$HW3\,V_{rms}$

1. By what factor does the absolute temperature change when the root mean square speed of the molecules is doubled?

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{T}\sqrt{\frac{3R}{M}}$$

$$2v_{rms} = \sqrt{4T}\sqrt{\frac{3R}{M}}$$

It changes by a factor of $\boxed{4}$

2. If the pressure in a gas is doubled while its volume is held constant, by what factor does Temperature change (assume the number of moles is held constant as well)?

If the pressure is doubled while the volume and the number of moles are held constant, the temperature doubling must be the cause.

B) By what factor does the root mean square speed of the molecules change?

$$v_{rms} = \sqrt{\frac{3R(2T)}{M}} = \sqrt{2}\sqrt{\frac{3RT}{M}}$$

It changes by a factor of $\boxed{\sqrt{2}}$

3. Two gases in different containers have the same concentration and the same root mean square speeds. The mass of a molecule of the first gas is twice the mass of a molecule of the second gas ($m_1 = 2m_2$). What is the ratio of the pressures of the two gases in their containers ($P_1/P_2 =?$)

$$v_1 = v_2$$

$$\sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}}$$

$$M_1 = 2M_2$$

$$\sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{2M_1}}$$

$$\sqrt{T_1} = \sqrt{\frac{T_2}{2}}$$

$$T_1 = \frac{T_2}{2}$$

$$\frac{T_1}{T_2} = \frac{1}{2}$$

The velocities of the molecules are the same, but because one set of molecules are twice as massive, that set of molecules has twice the kinetic energy, and thus, twice the temperature.

Thus, as pressure varies directly with temperature and as their concentrations (moles per liter) are equal, $\frac{P_1}{P_2} = \boxed{\frac{1}{2}}$

4. Two isotopes of uranium, U235 and U238, can be separated by a gas-diffusion process by combining them with fluorine (mass = 19 amu) to make the gaseous compound UF6. Calculate the ratio of the speeds ($U235F_6/U238F_6$) of these molecules for the two isotopes, at a constant temperature.

$$\frac{\sqrt{\frac{3RT}{M_{235}}}}{\sqrt{\frac{3RT}{M_{238}}}}$$

$$M_{235} = 349g/mol$$

$$M_{238} = 352g/mol$$

$$\frac{\sqrt{\frac{1}{349g/mol}}}{\sqrt{\frac{1}{352g/mol}}} = \sqrt{\frac{\frac{1}{349g/mol}}{\frac{1}{352g/mol}}} = \sqrt{\frac{352g/mol}{349g/mol}}$$

The ratio is $\boxed{1.004}$

5. What is the average molecular kinetic energy of the nitrogen (N_2) molecules in a sample (atomic number 7; atomic mass = 14 amu or 14 g/mol) at STP?

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3(8.315J/K/mol)(273K)}{0.028kg/mol}}$$

$$v_{rms} = 493.2m/s$$

$$K_e = \frac{1}{2}mv^2$$

$$K_e = \frac{1}{2}(28amu)(1.67 \times 10^{-27}kg/amu)(493.2m/s)^2$$

$$K_e = \boxed{5.687 \times 10^{-21}J}$$

B) What is the total internal (translational) kinetic energy of 1.00 mole of nitrogen molecules at $293K$?

$$E = \frac{3}{2}nRT$$

$$E = \frac{3}{2}(1mol)(8.315J/mol/K)(293K)$$

$$E = \boxed{3.656kJ}$$